

Spacetime Dimensionality from de Sitter Entropy

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We argue that the spontaneous creation of de Sitter universes favors three spatial dimensions. The conclusion relies on the causal-patch description of de Sitter space, where fiducial observers experience local thermal equilibrium up to a stretched horizon, on the holographic principle, and on some assumptions about the nature of gravity and the constituents of Hawking/Unruh radiation.

Introduction: The fact that the observable Universe has three large dimensions of space defies explanations other than anthropic [1]. This issue is sharpened by String Theory, which allows a humongous multitude of universes with various dimensions [2]. One may wonder whether the Anthropic Principle is the only way to understand the observed spacetime dimensionality.

The standard view of cosmology holds that the Universe began with an epoch of rapid expansion, called inflation [3], during which spacetime geometry was approximately de Sitter. In the braneworld context in String Theory, it was argued in [4] that the quantum creation of inflationary universes prefers one similar to our early Universe. String gas cosmology may also shed light on how only three (or less) spatial dimensions could have grown into a macroscopic size [5].

In this Letter we will argue, under certain fairly justifiable assumptions (to be spelled out as we proceed), that an inflationary cosmic origin implies three large dimensions of space. Our regime of interest for the possible number of spatial dimensions, d , is $2 \leq d \leq 10$. Such a restriction follows if one assumes that gravity is described by General Relativity in the infrared, and that the underlying theory of quantum gravity yields Supergravity as some low-energy approximation [30].

de Sitter Space & Entropy Thereof: We will use the natural units: $c = \hbar = k_B = G = 1$. In $(d+1)$ spacetime dimensions, this sets to unity the Planck length, $l_P \equiv \sqrt{d-1} \sqrt{\hbar c / 8\pi G}$, and the Planck mass, $M_P \equiv \hbar c^{-1} l_P^{-1}$, which however may be carried around for the sake of clarity.

Let us write down the $(d+1)$ -dimensional de Sitter metric in the *static* coordinates:

$$ds^2 = -(1 - H^2 r^2) dt^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega_{d-1}^2, \quad (1)$$

where $d\Omega_{d-1}^2$ is the line element on S^{d-1} , and the Hubble parameter H is related to the (positive) cosmological constant as: $\Lambda = \frac{1}{2}d(d-1)H^2$. The apparent singularity at $r = 1/H$ is a coordinate artifact. One can analytically extend the metric to a geodesically complete spacetime of constant curvature with topology $S^d \times \mathbb{R}^1$, where $r = 0$ represents antipodal origins of polar coordinates on a d -sphere. However, no single observer can access the entire de Sitter space: an observer at $r = 0$ experiences the presence of an event horizon at a distance $r = 1/H$. The “causal patch” of the observer is the region which is in full causal contact with her, namely $0 \leq r \leq 1/H$. The horizon is observer-dependent in that *any* observer following a time-like geodesic can be chosen to be at $r = 0$, and two such observers will belong to different causal patches. While the isometry group for de Sitter space is $SO(d+1, 1)$,

the manifest symmetries of the causal patch are $SO(d)$ rotations plus translation in t . The remaining d compact and d non-compact generators displace an observer from one causal patch to another.

In what follows we will restrict all attention to a *single* causal patch, à la [6, 7]. As regions that are out of causal contact with a particular observer have no operational meaning to her, the observer should consider the physics inside her horizon as complete, without making reference to any other region. Without loss of generality, this we can choose to be the “southern” causal patch, where the Killing vector ∂_t is time-like *and* future-directed, so that time evolution is well defined. We imagine that the causal patch is filled with “fiducial observers” (Fidos), each of whom is at rest relative to the static coordinate system, i.e. each is located at a fixed r and fixed values of the angular variables. The only geodesic observer is the Fido at $r = 0$, whom we call the “principal investigator” (PI). The PI can send a request to any other Fido to perform certain *local* measurements and report the results, which the PI will eventually receive after waiting for a finite amount of time.

As it is well known, the PI at $r = 0$ detects a thermal radiation with a temperature $T_{GH} = H/2\pi$ – the Gibbons-Hawking temperature of de Sitter space [8]. More generally, a Fido at a radial position r , whose Killing orbit has a proper acceleration $\alpha = H^2 r / \sqrt{1 - H^2 r^2}$, detects a thermal bath with an effective *local* temperature [9]:

$$T(r) = \frac{1}{2\pi} \sqrt{H^2 + \alpha^2} = \frac{H}{2\pi \sqrt{1 - H^2 r^2}}, \quad (2)$$

which is just the Gibbons-Hawking temperature multiplied by a Tolman factor [10]. Using an Unruh-like detector [11], the Fido can indeed discover a thermal radiation with the temperature $T(r)$. This effect is real, and can also be understood as pure Unruh effect associated with Rindler motion in the global embedding Minkowski space [12]. The local temperature, however, blows up at the horizon $r = 1/H$. A way to regularize this divergence is to consider a “stretched horizon” [13], that extends from the mathematical horizon (by some Planck length) up to some $r_c < 1/H$. The thickness may actually be a physical reality, originating possibly from quantum fluctuations [14]. The temperature measured at the stretched horizon is then large but finite:

$$T_c \equiv T(r = r_c) = \frac{H}{2\pi \sqrt{1 - H^2 r_c^2}} < \infty, \quad (3)$$

which sets a cutoff value for the temperature. It is natural to identify the cutoff with the Planck scale. In our case it will indeed turn out that $T_c \sim M_P$.

As was first pointed out in [10], a global notion of temperature may not be meaningful in curved space-times. Instead, one may need to introduce operationally meaningful local concepts of temperature and thermal equilibrium, as was advocated in [15], for example. It was shown in ref. [9] that the unique invariant *locally Minkowskian* state of quantum fields in de Sitter space has exactly the temperature given by (2). We will therefore consider only *local thermal equilibrium* with temperature $T(r)$ of the physical degrees of freedom (DOF) accessible to a Fido at a radial position r .

What DOFs does the thermal radiation contain, i.e. what are the constituents of Hawking/Unruh radiation? Postponing justification until later, we assume that the radiation is a gas of photons and gravitons, whose interactions are negligible. In $(d+1)$ spacetime dimensions photon has $(d-1)$ DOFs, while graviton has $\frac{1}{2}(d+1)(d-2)$. The total available DOFs is therefore

$$\mathfrak{D} \equiv \mathfrak{D}(d) = (d-1) + \frac{1}{2}(d+1)(d-2). \quad (4)$$

The PI can ask a Fido at a radial position r to measure the local entropy density, and receive the result

$$\sigma(r) = \left(\frac{d+1}{d} \right) \mathfrak{D} a(d) [T(r)]^d, \quad (5)$$

where $a(d)$ – the radiation constant per DOF in d spatial dimensions – is given by

$$a(d) = \frac{\omega(d)}{(2\pi)^d} \zeta(d+1) \Gamma(d+1), \quad \omega(d) \equiv \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})}, \quad (6)$$

$\omega(d)$ being the surface area of the boundary of a unit d -ball. The PI can take the volume integral of (5) to compute the total thermal entropy of the causal patch:

$$S = \omega(d) \int_0^{r_c} \frac{dr r^{d-1}}{\sqrt{1-H^2 r^2}} \sigma(r). \quad (7)$$

One can express this integral in terms of hypergeometric functions by first making the variable redefinition: $x \equiv H^2 r^2$, and then using the integral representation [16]:

$$\int_0^z \frac{dx x^{\alpha-1}}{(1-x)^{1-\beta}} = \frac{z^\alpha}{\alpha} (1-z)^\beta {}_2F_1(\alpha+\beta, 1; \alpha+1; z), \quad (8)$$

which holds for $\Re(\alpha) > 0$. Thereby the PI finds that the total entropy amounts to

$$S = \left(\frac{d+1}{d^2} \right) \mathfrak{D} \omega(d) a(d) \left(\frac{\sqrt{1-\epsilon}}{2\pi} \right)^d \left(\frac{1}{\sqrt{\epsilon}} \right)^{d-1} \times {}_2F_1\left(\frac{1}{2}, 1; \frac{d+2}{2}; 1-\epsilon\right), \quad (9)$$

where ϵ is a positive number defined as

$$\epsilon \equiv 1 - H^2 r_c^2 = \left(\frac{H}{2\pi T_c} \right)^2. \quad (10)$$

Note that $\epsilon \ll 1$, because we consider $H/2\pi$ to be much smaller than $T_c \sim M_P$, which is necessary for a semi-classical treatment to be valid. The total entropy (9), which clearly diverges in the limit $\epsilon \rightarrow 0$, is rendered large but finite by the stretched horizon.

Now, there is an entropy associated with de Sitter horizon, known as the Gibbons-Hawking entropy [8], which is $\frac{1}{4}$ of the horizon area, A , in Planck units:

$$S_{\text{GH}} = \frac{1}{4} \frac{A}{l_P^{d-1}} = \frac{1}{4} \omega(d) \left(\frac{M_P}{H} \right)^{d-1}. \quad (11)$$

To see its possible connection with the total entropy (9) of the causal patch, we note that the hypergeometric function appearing in the latter can be written as [16]:

$${}_2F_1\left(\frac{1}{2}, 1; \frac{d+2}{2}; 1-\epsilon\right) \equiv \left(\frac{d}{d-1} \right) [1 + \delta(d, \epsilon)], \quad (12)$$

where the function δ is such that it vanishes in the limit $\epsilon \rightarrow 0$. Thanks to Eq. (12) and some properties of the gamma function, one can rewrite the entropy (9) as

$$S = \omega(d) \left[\frac{\mathfrak{D} \Gamma(\frac{d+3}{2}) \zeta(d+1)}{(d-1)(\sqrt{\pi})^{d+3}} \right] \left(\frac{T_c}{H} \right)^{d-1} + \dots, \quad (13)$$

where the ellipses stand for subleading terms. Their H -dependencies differ from that of the leading term, which actually mimics the area law (11) for de Sitter entropy.

Now we invoke the holographic principle, which entails that the leading term in (13) should be identified with the Gibbons-Hawking entropy (11) of the de Sitter horizon [17, 18]. This relates the cutoff temperature T_c and the Planck mass M_P in the following way [19]:

$$\frac{T_c}{M_P} = \left[\frac{(d-1)(\sqrt{\pi})^{d+3}}{4 \mathfrak{D} \Gamma(\frac{d+3}{2}) \zeta(d+1)} \right]^{\frac{1}{d-1}}. \quad (14)$$

Note that the cutoff T_c is independent of the Hubble parameter H , as expected, but depends on the number of DOFs in a way that is in complete accordance with the results of [20]. With \mathfrak{D} given by Eq. (4), the right-hand side of Eq. (14) is $\mathcal{O}(1)$. This sets $T_c \sim M_P$. Then, it is easy to see that the thickness of the stretched horizon is $\mathcal{O}(l_P)$. With the identification (14), one can now make explicit the area dependence of the entropy:

$$S = \frac{A}{4} \left(\frac{d-1}{d} \right) (1-\epsilon)^{d/2} {}_2F_1\left(\frac{1}{2}, 1; \frac{d+2}{2}; 1-\epsilon\right), \quad (15)$$

where the parameter ϵ depends on A as follows

$$\epsilon = \frac{1}{4\pi^2} \left[\frac{8 \mathfrak{D} \Gamma(\frac{d+3}{2}) \zeta(d+1)}{A (d-1) \pi^{3/2} \Gamma(\frac{d}{2})} \right]^{\frac{2}{d-1}}. \quad (16)$$

In the limit $\epsilon \rightarrow 0$ or $A \rightarrow \infty$, thanks to Eq. (12), S/A reaches the value $\frac{1}{4}$ for *any* space dimensionality.

Similarly, the PI can define the total “energy” of the causal patch as follows.

$$E = \omega(d) \int_0^{r_c} \frac{dr r^{d-1}}{\sqrt{1-H^2 r^2}} \rho(r), \quad (17)$$

where $\rho(r) = \mathfrak{D} a(d) [T(r)]^{d+1}$ is the local energy density that a Fido at a radial position r reports to the PI. The total “energy” turns out to be

$$E = \frac{A}{4} \left(\frac{d-1}{d+1} \right) (1-\epsilon)^{d/2} \left[\frac{(d-1)(\sqrt{\pi})^{d+3}}{4 \mathfrak{D} \Gamma(\frac{d+3}{2}) \zeta(d+1)} \right]^{\frac{1}{d-1}}, \quad (18)$$

where we have made use of Eqs. (8), (10) and (14). As the total “energy” (18) follows an area law, just like the total entropy does [21], it is natural to consider also E as an attribute of the de Sitter horizon.

Here we are not calling for a thermodynamic description of the de Sitter horizon [8, 22, 23] (whose validity is yet to be well established anyway). Therefore, our total “energy” E should not be confused with what is proposed in [22], for example, as the energy associated with a de Sitter horizon, which is *not* proportional to the horizon area. The former quantity is unique and well defined in the following sense. As soon as the de Sitter entropy (15) is taken to be finite, we must forgo the symmetry of different causal patches [7]. In the *given* causal patch, all the Fidos are on equal footing in that they all follow time-like trajectories, are in causal contact with one another, and of course experience the same causal horizon. Any quantity to be attributed to the entire causal patch or to the horizon itself must not depend on which Fido, be her the PI or not, is assigned the job of defining it. In other words, all the Fidos must agree upon the value of any such quantity. Now that any Fido can learn about the results of local measurements performed by any other Fido, they all will have identical sets of data for the density distributions $\sigma(r)$ and $\rho(r)$, and therefore will agree upon their respective volume integrals S and E . This is not the case if in the definition (17) one inserts a redshift factor (as was suggested in the ref. in [21]), which itself depends on the position of the Fido assigned to define the quantity.

Eqs. (15) and (18) can be viewed as relations among three extensive properties of the horizon, namely S , E and A . Dividing Eq. (15) by (18), one can also write

$$\frac{S}{E} = \left(\frac{d+1}{d} \right) \left[\frac{4 \mathfrak{D} \Gamma \left(\frac{d+3}{2} \right) \zeta(d+1)}{(d-1)(\sqrt{\pi})^{d+3}} \right]^{\frac{1}{d-1}} \times {}_2F_1 \left(\frac{1}{2}, 1; \frac{d+2}{2}; 1-\epsilon \right). \quad (19)$$

It is clear, in view of Eq. (12), that in the limit $\epsilon \rightarrow 0$, the ratio S/E is finite, although both S and E diverge. For $\epsilon \ll 1$, Eq. (19) can be approximated as

$$S \approx E \left(\frac{d+1}{d-1} \right) \left[\frac{4 \mathfrak{D} \Gamma \left(\frac{d+3}{2} \right) \zeta(d+1)}{(d-1)(\sqrt{\pi})^{d+3}} \right]^{\frac{1}{d-1}}. \quad (20)$$

The virtue of Eq. (20) is that it allows one to compare the entropies of de Sitter spaces with different d , while keeping E fixed. It does not make sense to compare the horizon areas of two spaces with different dimensionalities, and indeed A does not appear in (20).

Is 4D Spacetime Natural? Let us consider the creation of de Sitter universes, by quantum tunneling [24], with different spacetime dimensionalities, but with the same values of the fundamental constants, which all can be set to unity [31]. One could estimate the tunneling probability amplitude via the Hartle-Hawking wavefunction [25] of the de Sitter universe [32]:

$$\Psi_{\text{HH}} \sim \exp(-\Gamma) = \exp(S), \quad (21)$$

where the Euclidean action, Γ , is simply the negative of the de Sitter entropy [26]. The larger the entropy,

the more probable is the tunneling event. Now for any *given* value of the characteristic quantity E , one can use Eq. (20) to formally consider S as a function of the number of space dimensions d . Is there any particular value of d that maximizes the probability amplitude (21)?

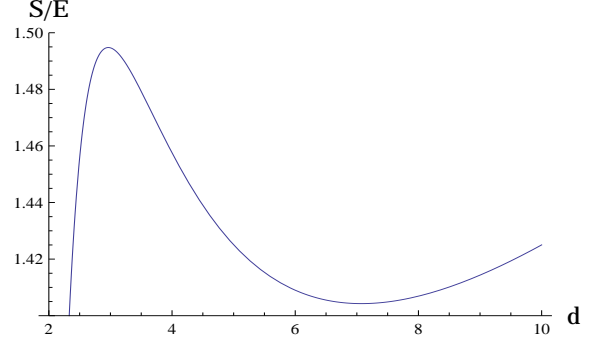


FIG. 1: de Sitter entropy versus space dimensionality

Fig. 1 plots S/E as a function of d , which is treated as a continuous variable in the regime $2 \leq d \leq 10$. We notice an absolute maximum at $d \approx 2.97$. An upper bound on d is essential for the result, since for large d , the function increases monotonically: $S/E \sim \sqrt{d/2\pi e}$. The values of S/E are $\mathcal{O}(1)$. Explicitly, the respective values for $d = 2, 3, 4, \dots, 10$, are approximately 1.096, 1.495, 1.458, 1.425, 1.409, 1.404, 1.407, 1.414, 1.425.

The characteristic quantity E , however, is very large; typically, one may consider $E \sim S \gtrsim 10^{10}$. A tiny difference in the value of S/E therefore corresponds to a very big difference in S . Then, the probability amplitude for $d = 3$ is larger than that for any other dimension by a huge margin. In other words, the creation of a (3+1)D de Sitter universe is overwhelmingly more likely.

Remarks: The shape of the function S/E crucially depends on what DOFs are taken into account. Thus, one might turn the argument around to conclude that the constituents of Hawking/Unruh radiation must be photons and gravitons, if the creation of a (3+1)D de Sitter universe is most probable. However, one may also justify why it is sensible to consider only those DOFs.

First, note that the radiation should comprise only particles with mass $m \ll H$; otherwise, it will not be (approximately) thermal, and the density distributions $\sigma(r)$ and $\rho(r)$ will not be smooth. Suppose there is a particle with a mass $m' \neq 0$, which, if the Hubble parameter instead was $H' \ll m'$, would not contribute to the total number of DOFs \mathfrak{D} . Then Eq. (14) means that the cutoff T_c would be sensitive to the Hubble parameter, which is unacceptable. The conclusion is that the constituent particles should all be *strictly* massless. Known difficulties with massless higher spins [27] then make it natural to consider only particles of spin $s \leq 2$. If there is a massless spin-1 particle, no other particle can be charged under this, because otherwise interactions would render the radiation non-thermal. While the Hawking/Unruh effect is very fundamental and takes place in all dimensions, a massless chargeless fermion can exist only in some particular dimensions. This rules out spin-1/2 and spin-3/2 particles as non-generic. Scalars are also ruled out, since there is no symmetry to assure their masslessness. So we are left only with spin-1 and spin-2 particles, whose masslessness can be guaranteed by gauge invariance. Now, there

is one and just one massless spin 2, namely the graviton [27]. More than one vector particle is not a possibility, because they either confine and cease to exist as long-range particles (when they are mutually charged), or there is no way to distinguish them as different constituent particles of the radiation. At any rate, that photons and gravitons could be the sole constituents of the radiation may not seem so surprising given that these are the natural DOFs to consider at low energy.

It is curious to notice that even when d is considered as continuous-valued, we get to the result of $d = 3$ up to 1% accuracy. While it is easy to convince oneself that the finiteness of ϵ has little affect on this result, it remains to be seen how things change when the very weak interactions between photons and gravitons, which we have neglected in this Letter, is taken into account.

This Letter did not address the issue of low-entropy origin for our Universe, nor did it present any particular framework in which inflation is one among various

competing cosmological scenarios, whose relative probabilities of creation can be quantified [6, 28]. Rather, by relying on some fairly justifiable assumptions, and not on the details of an underlying quantum gravity theory, we argued that the spontaneous creation of inflationary universes favors three dimensions of space. Whether or not inflation is the correct scenario for cosmic origin, it is tempting to ask if such arguments are of any relevance to the late phase of our Universe, which is asymptotically de Sitter [29] with three large spatial dimensions.

Acknowledgments: We are thankful to G. Gibbons, C. Hull, A. Iglesias, E. Joung, M. Kleban, D. Klemm, L. Lopez, M. Porrati, A. Sagnotti, L. Sorbo and M. Taronna for useful discussions. AM would like to thank for the hospitality of the HECAP section of ASICTP, Trieste, where part of this work was done. RR was supported in part by Scuola Normale Superiore, by INFN and by the ERC Advanced Investigator Grant no. 226455 “Supersymmetry, Quantum Gravity and Gauge Fields” (SUPERFIELDS).

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